**Chapter 5: Exponential and Logarithmic Functions**

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**Chapter 5.1: Math Lab: Graphing Exponential Functions.**

**Focus:** Estimate the graphs of exponential Functions.

Evaluate each Power

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Graph using a table of values the following

How are the above functions alike? How are they different?

The graphs of both functions have the same shape but a) increases as x increases, while b) decreases as z increases. Both graphs have y – int =1, no x –int, and H.A. is y = 0 .

**Chapter 5.2: Analyzing Exponential Functions**

**Focus:** Sketch the graph of an exponential function and describe its characteristics.

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| **Exponential Function**An exponential function is any function of x that can be written in the form , where a is a positive constant. |

Graph a general form of when x > 1 and when 0 < x < 1 , then outline the characteristics of this graph. (y – int, x – int, H.A. Domain and range, behaviour at the extremes.)

Pg 345

## Example 1: Sketching the Graph of an Exponential Function and identifying its characteristics.

Graph , and determine

* Effect on y when x increases by 1
* Whether the function id increasing or decreasing
* The intercepts
* Asymptotes
* Domain and range.

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| **The Function** **Relate this to the graph of**   |

## Example 2: Transforming the graph of

Use the graph of to sketch the graph of . Determine whether the function is increasing or decreasing, the intercepts, asymptotes, domain and range…(the characteristics)

## Example 3: Using transformations of Exponential Functions to Model Real-World Situations

For every metre below the surface of water, the light intensity is reduced by 2.5%. The percent, P of light remaining at a depth d metres can be modelled by the function

Graph the function for

How much light remains at a depth of 10 m.

What is the depth when only 50% of the light remains.

**Chapter 5.3: Solving Exponential Equations.**

Focus: Solve problems by modelling situations with exponential equations.

Skill Check: Solve algebraically. How would you do this graphically?

Use the same two strategies to solve .

An exponential equation contains a power with a variable in the exponent. One strategy to solve these types of equations is to use the fact that when two powers with the same base are equal, their exponents are also equal. Mr. Hogg described this method in previous years as ‘cheating’ or ‘magic’.

## Example 1: Solving an exponential Equation using common bases.

Solve

## Example 2: Solving an Exponential Equations Involving a Radical.

Suppose you have $100 and you invest it in a variety of accounts. All of the accounts earn you 2% interest.

The first account, interest is compounded annually. Therefor at the end of one year you will have , after two years you have .

How much will you earn after three years? Ten years?

The second account, interest is compounded **semi-annually**; this means that the annual rate is halved and the number of compounding periods is doubles. Therefore at the end of one year you will have

, where t is the number of years…in this case 1.

At the end of two years. , t = 2

How much money is earned after 3 years? After 10 years?

The third account interest is compounded quarterly. The interest rate is one quarter of the annual rate and there are four compounding periods per year.

At the end of two years , is earned. Where t is the number of years the account is active.

How much money will this account earn after 3 years? After ten years?

The last account is compounded every month. This is often called monthly compounding, the amount earned after 3 years is

$ earned after three years? Ten years?

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| Effects of compounding Interest rates. calculations on an initial investment of $100 with a interest rate of 2% |
| Interest Rate Calculations | Annually | Semi-Annually | Quarterly | Monthly |
| One year |  |  |  |  |
| Three Year |  |  |  |  |
| Ten Year |  |  |  |  |

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| Compound Interest A principal of A0 dollars is invested at an annual interest rate of *I*, with *n*, compounding periods per yeare. The amount, A dollars, after *t* years is given by : Where is known as the Growth Factor. |

Compund interest is an example of Exponential Growth.



## Example Three: Solving a Problem Involving Exponential Growth

A principal of $1500 is invested at 4% annual interest, compounded quarterly. To the nearest quarter of a year, when will the amount be $2500?



# Example Four: Solving a problem involving Exponential Decay

IF the cabin pressure in an airplane is less than 70 kPa, passengers can suffer altitude sickeness. To the nearest km, at what altitude is the atmospheric pressure 70 kPa. Air Pressure is modelled by the function where *h* is the height in Km and P is measured in kilo Pascals.

CheckPoint

# 5.4 Logarithms and the Logarithmic Function

Focus: Investigate logarithmic functions and relate them to exponential functions.

Us a table of values and graph both



Now let’s graph the function ,

What’s the domain and Range, asymptote? And what could the inverse be?

I Looks like has an inverse. To describe this inverse we need another term.

Logarithm: is used to describe the inverse of a power.

So the inverse of will look something like .



## Example One: Writing Expressions in Different Terms

Write each exponential expression as a logarithm.

1. Write each logarithmic ezpression as an exponential expression
	1.

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| More logarithm DefinitionsIf; , Then; and are inverses.These can be used to simplify expression involving exponents and logs (logarithms) |

# Example Two: Evaluation Logarithms

# Example 3: Using Benchmarks to Estimate the value of a logarithm

To the nearest tenth estimate the value of

# Example 4: Identifying the characteristics of the Graph of a logarithmic function

1. Graph
2. Identify the intercepts, asymptotes, domain and range.



**Chapter 5.5: Laws of Logarithms**

Focus: Develop and use the laws of logarithms.



The definithio of a logarithm can be used to prove that the laws above are true for all logarithms.

Let’s prove the Product law: To prove that

Let

# Example One: Applying the Laws of Logarithms to Logarthms with Base 10.

Simplify each expression. Use a calculator to verify the answer

# Exampel Two: Using the Laws of Logarithms to Simplify Expressions.

# Example Three: Writing a Logarithm as a Sum or Difference of Logarithms

Write each expression in terms of log a, log b , and or log c

# Example Four: Evaluate using Laws of Logarithms.

**Chapter 5.6: Analyzing Logarithmic Functions**

Focus: Use technology to graph transformations of logarithmic functions.

Use this super amazing trick to evaluate logarithms.



# Example One: Use Tech to Approximate the value of a logarithm.

Approximate the following to the nearest thousandth. Writhe the related exponential expression.

# Example Two: Use tech to graph a logarithmic function.

Graph

Describe the characteristics of the graph

# Example Three: Transforming the Graph of a Logarithmic Function.

 Create a table of values for

 How is the graph of is related to the graph of . Sketch these two graphs on the same grid.

 , Describe the characteristics.

**Chapter 5.7: Logarithmic and Exponential Equations**

Focus: Solve logarithmic and exponential equations algebraically.

Simplify each expression.

Let’s use algebra to solve the following and ‘verify’ the solution.

A logarithmic equation that contains the logarithm of a variable. The laws of logarithms can be used to solve logarithmic equations.

# Example one: Solving a Logarithmic Equation involving

Solve and verify:

# Example two: Solving a logarithmic Equation involving

Solve, then verify;

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# Example three: Using Logarithms to Solve Exponential Equations

Solve algebraically. Solution to nearest hundredth.

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In Example 3c, can a different base be used? Do you have to use common logarithms?

**Chapter 5.8:Solving Problems with Exponents and Logarithms**

Focus: Use exponents and logarithms to model and solve problems.

Use the formula for the sum of a geometric series to determine the sum of the series: , this is practice in calculator use.

Let’s see if we can use our new knowledge of logarithms to do an old question type.

A principal of $5000 is invested at 3% annual interest, compounded monthly. Use algebra to determine the time, to the nearest year, it will take for the investment to double. Graph to verify.

When a series of equal investments is made at equal time intervals, and the compounding period for the interest is equal to the time interval for the investments, the amount in dollars, or FUTURE VALUE FV, of these investments can be determined using this formula:

Where R –

 I –

 n –

# Example one: Solving a problem involving future value.

Determine how many monthly investments of $200 would have to be made into an account that pays 6% annual interest, compounded monthly, for the future value to be $100 000.

What would happen if 180 monthly investments of $100 were made into the account? Would the futre value be $50 000?

Many people borrow money to finance a purchase…how many \_\_\_ financial decisions have you seen around Mackenzie? A loan is usually repaid by making regular equal payments for a fixed period of time. The amount borrowed is classed the present value, PV of the loan. The following formula relates the present value to *n* equal payments of *R* dollars each, when the interest rate per compounding period is *i*. The compounding period is equal to the time between payments. The first payment is made after a time equal to the compounding period.

Example Two: Solving a problem involving loans

A person borrows $15 000 to buy a car. The person can afford to pay $300 a month. The loan will be repaid with equal monthly payments at 6% annual interest, compounded monthly. How many monthly payments will the person make?

\*\*\* When physical quantities have a large range of values, they can be measured using a logarithmic scale. Some examples include the Richter scale, the decibel scale, and the pH scale.

The magnitude M of and earthquake is defined as

The intensity of the vibrations of an earthquake, I microns, is measured on a seismograph that is 100 km away from the epicentre of the earthquake.

This intensity is compared to the intensity, S of a standard earthquake. Which has a seismograph reading of 1 micron and can barely be detected.

The righter scale. Each increase of 1 unit on this logarithmic scale represents a 10-fold increase in intensity. For example an earthquake of magnitude 9 is 100 time larger than an EQ of magnitude 7.

# Example Three: Solving a problem involving the Richter scale.

The most intense earthquake ever recorded was in Chile in May 1960, with a magnitude of 9.5.

Calculate the intensity of the EQ in Chile in terms of a standard EQ.

How many times as intense as the Haiti (7.0) EQ was the Chile EQ?